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## Equations of Hyperbolas - Guided Lesson Explanation

Hyperbolas have an equation that fits the model:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

(h, k) = center a= semi-tranverse axis b = semi-conjugate axis

**Explanation#1**

If we look at the foci, we will see that they are side-by-side. This indicates that branches of the hyperbola follow this lead. This also means that the center, foci, and vertices are on a line that is parallel to the x-axis.

We need to remind ourselves a few things:

a. The center is in the middle of the foci. In this case, the center must be at (h, k) = (–1, 0).

b. The foci are offset from the center. In this problem the foci are 6 units to either side of the center. So $c = 6$ and $c^2 = 36$.

c. The center resides on the x-axis. This means that the x-intercepts have to also be vertices for the hyperbola. The intercepts are offset from the center and are 5 units in either direction of the center.

$a = 5$ and $a^2 = 25$.

Then $a^2 + b^2 = c^2$ tells us that $b^2 = 36 - 25 = 11$, and the equation is:

$$\frac{(x+1)^2}{25} - \frac{y^2}{11} = 1$$

**Explanation#2**

We can see in this problem that all three measures are on the horizontal line $y = 7$.

The vertex is 3 units from the center, so that $a = 3$

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The focus is 7 units from the center, so that $c = 7$.

Then $a^2 + b^2 = c^2$ gives me $b^2 = 49 - 9 = 40$.

Putting it all together, the equation would look like this:

Final) $$\frac{(x-3)^2}{9} - \frac{(y-7)^2}{40} = 1$$

**Explanation#3**

If we look at the foci, we will see that they are side-by-side. This indicates that branches of the hyperbola follow this lead. This also means that the center, foci, and vertices are on a line that is parallel to the x-axis.

We need to remind ourselves a few things:

a. The center is in the middle of the foci. In this case, the center must be at (h, k) = (–1, 0).

b. The foci are offset from the center. In this problem the foci are 8 units to either side of the center, so $c = 4$ and $c^2 = 16$.

c. The center resides on the x-axis. This means that the x-intercepts have to also be vertices for the hyperbola. The intercepts are offset from the center and are 2 units in either direction of the center.

$a = 2$ and $a^2 = 4$.

Then $a^2 + b^2 = c^2$ tells us that $b^2 = 16 - 4 = 12$, and the equation is:

Final) $$\frac{(x+1)^2}{4} - \frac{y^2}{12} = 1$$