

## Equations of Ellipses - Step-by-Step Lesson

Find the equations of an ellipse (in standard form) that has foci at (0,-5) and (0,5) and vertices at (0,-11) and (0,11).

### Explanation:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

To write the equation in standard form, the center,  $a^2$  and  $b^2$  must be found. The x coordinate of the vertices and foci given are 0, so the center of the ellipse has to be (0,0).

$|a|$  = horizontal distance

$|b|$  = vertical distance

(H, k) = Center

(0,0) = Center

Vertices: (0,-11), (0,11)

Vertices are 22 units apart.

Foci: (0,5), (0,-5)

Foci are 10 units apart.

Since the foci are 10 units apart, indicating that  $|c|$  is 5 and  $c^2$ , then is 25.

$$|c| = 5 \quad c^2 = 5^2$$

To find  $b^2$ , use the foci equation.  $c$  is the distance from the center of the ellipse to the foci. Since the center is (0, 0), and the vertices are (0,-11) and (0, 11),  $a$  equals 11.

$$c^2 = |a - b|$$

$|c|$  = distance from center to foci



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(0,0)=Center

(0,-11),(0,11)= vertices

|a| = distance from center to vertex

|a| = 11

Now that we have the values for a and c, we can substitute the values into the equation and simplify to find b.

$$c^2 = |a^2 - b^2|$$

$$(5)^2 = |11^2 - (b)^2|$$

$$25 = |121 - b^2|$$

$$b^2 = 121 - 25$$

$$b^2 = 96$$

Substitute 0 for h and k, the square root of 96 for b and 11 for a into the standard form equation of an ellipse. The equation is y squared over the square of 11 plus x squared over the square of the square root of 96 equals to 1.

$$\frac{(y-h)^2}{a^2} + \frac{(x-k)^2}{b^2} = 1$$

$$\frac{(y-0)^2}{(11)^2} + \frac{(x-0)^2}{(\sqrt{96})^2} = 1$$

$$\frac{y^2}{(11)^2} + \frac{x^2}{(\sqrt{96})^2} = 1$$

