Models of Data - Guided Lesson Explanation

For all the problems, we can compare the successive y-values.

Linear Functions = The first differences between successive y-values are equal. They make a line.

Quadratic Functions = The second differences between successive y-values are equal.   They make a parabola when graphed.

Exponential Functions = If the ratios between successive y-values are equal. They get steeper and steeper as they go along.

Explanation#1

Find the first differences in the table.

x	y
0	-4
1	0
2	8
3	20
4	36

+4

+8

+12

+16

Find the second differences in the table.

x	y
0	-4
1	0
2	8
3	20
4	36

+4   +4

+8   +4

+12   +4

+16

Since the second differences are all equal, the function is quadratic.

$(y = ax^2 + bx + c)$ Because the function is quadratic it takes this form.

a term = rate of the rate = 4/2 = 2

c term = y-intercept = -4

The b term is a little more tricky. We start by looking at what we already have and plug our given x values in. This will tell us how to determine b.

$2x^2$ -4

x	y	1) y-value without b	2) value missing
0	-4	-4	0
1	0	-2	-2
2	8	4	-4
3	20	14	-6
4	36	28	-8

In 1, we calculate the value without b inserted in the equation. In 2, we see how far the value is off from the actual value with b present.

Now we look at 2 and determine how far off each value is.

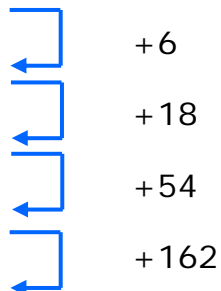They are off by -2. This means that b is -2x. Now we put it together for the final euqation.
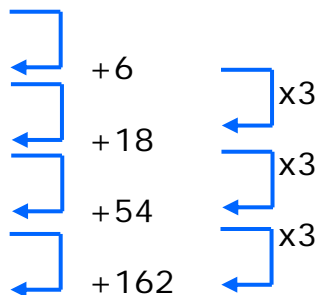
$2x^2$ -2x -4

Explanation#2

Find the first differences in the table.

x	y
1	5
2	11
3	29
4	83
5	245

+6

+18

+54

+162

Since the differences are not all equal, the function can't be quadratic. Find the second differences in the table.

x	y
1	5
2	11
3	29
4	83
5	245

+6

+18

+54

+162

x3

x3

x3

The ratios between successive y-values are all 3, so the function is exponential ($y=a^x$). When the ratios are the same, the base a is equal to the common ratio, 3.

$(y = 3^x)$

Now we need to see if anything further is being done to the data beyond ($y = 3^x$).

Current values   Values using y = 3^x

x	y
1	5
2	11
3	29
4	83
5	245

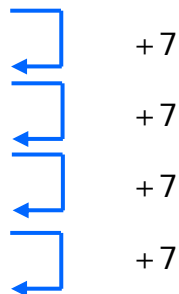| x	y	difference between tables
1	3	-2
2	9	-2
3	27	-2
4	81	-2
5	243	-2

To make the values equal we would need to add 2.

The finally equation would need to be $3^x + 2$.

Explanation#3

Find the first differences in the table.

x	y
-2	9
-1	16
0	23
1	30
2	37

+7

+7

+7

+7

Since the first differences are all equal, the function is linear(y = mx + b).

(y = mx + b)

The slope is "m".

The starting point is "b" or x = 0. The y-value is 23.

Now, plug in one of the points from the table.

y = 7(-2) + 23)

The function is y = 7(-2) + 23).