

Finding the Inverse of a Matrix - Guided Lesson Explanation

Explanation#1

Step 1) First we will see what is being asked.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 5 & 6 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-5R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & -4 & -15 & -5 & 0 & 1 \end{array} \right)$$

Step 2)

$$4R_2 + R_3 \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array} \right) \quad -2R_2 + R_1 \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -5 & 1 & -2 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array} \right)$$

Step 3)

$$\begin{array}{l} 5R_3 + R_1 \\ -4R_3 + R_2 \end{array} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -24 & 18 & 5 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array} \right)$$

So, the inverse matrix is:

$$A^{-1} = \left(\begin{array}{ccc|ccc} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{array} \right)$$

Explanation#2

Step 1) First we will see what is being asked.

$$\left(\begin{array}{ccc} 1 & 0 & -3 \\ 2 & -2 & 1 \\ 0 & -1 & 3 \end{array} \right)$$

Step 2)

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array} \right)$$



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Step 3) Subtract two times the first row from the second row:

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & -2 & 7 & -2 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array} \right)$$

Step 4) Add the second and third rows:

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 1 \\ 0 & -2 & 7 & -2 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array} \right)$$

Step 5) Swap the second and third rows and subtract two times the second row from the third row:

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 6 & -3 & 7 \\ 0 & 0 & 1 & -2 & 1 & -2 \end{array} \right)$$

Step 6) Multiply the second row by -1.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 3 & -6 \\ 0 & 1 & 0 & -6 & 3 & -7 \\ 0 & 0 & 1 & -2 & 1 & -2 \end{array} \right)$$

So, the answer is :

$$\left(\begin{array}{ccc} -5 & 3 & -6 \\ -6 & 3 & -7 \\ -2 & 1 & -2 \end{array} \right)$$

Explanation#3

Step 1) First we will see what is being asked.

$$\left(\begin{array}{ccc} 2 & -1 & 0 \\ 1 & 3 & -1 \\ -3 & 0 & 1 \end{array} \right)$$



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Step 2)

$$\begin{pmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ 1 & 3 & -1 & 0 & 1 & 0 \\ -3 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Step 3) Add the third row to the second row:

$$\begin{pmatrix} 2 & -1 & 0 & | & 1 & 0 & 0 \\ -2 & 3 & 0 & | & 0 & 1 & 0 \\ -3 & 0 & 1 & | & 0 & 0 & 1 \end{pmatrix}$$

Step 4) Add the first row to the second rows:

$$\begin{pmatrix} 2 & -1 & 0 & | & 1 & 0 & 1 \\ 0 & 2 & 0 & | & 1 & 1 & 1 \\ -3 & 0 & 1 & | & 0 & 0 & 1 \end{pmatrix}$$

Step 5) Add half the second row to the first row:

$$\begin{pmatrix} 2 & 0 & 0 & | & 1.5 & 0.5 & 0.5 \\ 0 & 2 & 0 & | & 1 & 1 & 1 \\ -3 & 0 & 1 & | & 2.25 & 0.75 & 1.75 \end{pmatrix}$$

Step 6) Multiply the first and second rows by 0.5:

$$\begin{pmatrix} 1 & 0 & 0 & | & 0.75 & 0.25 & 0.25 \\ 0 & 1 & 0 & | & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 1 & | & 2.25 & 0.75 & 1.75 \end{pmatrix}$$

So, the answer is

$$\begin{pmatrix} 0.75 & 0.25 & 0.25 \\ 0.5 & 0.5 & 0.5 \\ 2.25 & 0.75 & 1.75 \end{pmatrix}$$

