## Equations of Ellipses - Guided Lesson Explanation

## Explanation\#1

Step1) First we should see what we have find out.
Step2) $\frac{(x-h)^{2}}{a^{2}}+\frac{(y-k)^{2}}{b^{2}}$
To write the equation in standard form, the center, $a^{2}$ and $b^{2}$ must be found. The $x$ coordinate of the vertices and foci given are 0 , so the center of the ellipse has to be $(0,0)$.
$|\mathrm{a}|=$ horizontal distance
|b| = vertical distance
$(\mathrm{H}, \mathrm{k})=$ Center
$(0,0)=$ Center
Vertices: (0,-13), $(0,13)$
Vertices are 26 units apart.
Foci: $(0,6),(0,-6)$
Foci are 12 units apart.
Since the foci are 12 units apart, indicating that $|c|$ is 6 and $\mathrm{c}^{2}$, then is 36 .
$|c|=6 \quad c^{2}=6^{2}$
To find $b^{2}$, use the foci equation. $c$ is the distance from the center of the ellipse to the foci. Since the center is ( 0,0 ), and the vertices are ( $0,-13$ ) and ( 0,13 ), a equals 13 .
$c^{2}=|a-b|$
$|\mathrm{C}|=$ distance from center to foci
$(0,0)=$ Center
$(0,-13),(0,13)=$ vertices
$\qquad$
|a| = distance from center to vertex
$|a|=13$
Now that we have the values for a and c, we can substitute the values into the equation and simplify to find $b$.

$$
\begin{aligned}
& \mathrm{C}^{2}=\mid a^{2}-b^{2} \\
& (6)^{2}=\left|13^{2}-(b)^{2}\right| \\
& 36=\left|169-b^{2}\right| \\
& \mathrm{b}^{2}=169-36 \\
& \mathrm{~b}^{2}=133
\end{aligned}
$$

Substitute 0 for $h$ and $k$, the square root of 133 for $b$ and 13 for a into the standard form equation of an ellipse. The equation is $y$ squared over the square of 13 plus $x$ squared over the square of the square root of 133 equals tol.

$$
\begin{aligned}
& \frac{(y-h)^{2}}{a^{2}}+\frac{(x-k)^{2}}{b^{2}}=1 \\
& \frac{(y-0)^{2}}{(13)^{2}}+\frac{(x-0)^{2}}{(\sqrt{133})^{2}}=1 \\
& \frac{y^{2}}{(13)^{2}}+\frac{x^{2}}{(\sqrt{133})^{2}}=1
\end{aligned}
$$

## Explanation\#2

Step1) First we should see what we have find out.
Step2) $\frac{(x-h)^{2}}{a^{2}}+\frac{(y-k)^{2}}{b^{2}}$
$\qquad$

To write the equation in standard form, the center, $\mathrm{a}^{2}$ and $\mathrm{b}^{2}$ must be found. The $x$ coordinate of the vertices and foci given are 0 , so the center of the ellipse has to be $(0,0)$.
|a| = horizontal distance
|b| = vertical distance
$(\mathrm{H}, \mathrm{k})=$ Center
$(0,0)=$ Center
Vertices: $(0,-15),(0,15)$
Vertices are 30 units apart.
Foci: $(0,9),(0,-9)$
Foci are 18 units apart.
Since the foci are 10 units apart, indicating that $|c|$ is 9 and $c^{2}$, then is 81 .
$|c|=9 \quad c^{2}=9^{2}$
To find $b^{2}$, use the foci equation. $c$ is the distance from the center of the ellipse to the foci. Since the center is ( 0,0 ), and the vertices are ( $0,-15$ ) and ( 0,15 ), a equals 15 .
$c^{2}=|a-b|$
$|C|=$ distance from center to foci
$(0,0)=$ Center
$(0,-15),(0,15)=$ vertices
|a| = distance from center to vertex
$|\mathrm{a}|=15$
Now that we have the values for a and c, we can substitute the values into the equation and simplify to find $b$.

$$
\begin{aligned}
& C^{2}=\left|a^{2}-b^{2}\right| \\
& (9)^{2}=\left|15^{2}-(b)^{2}\right|
\end{aligned}
$$

$\qquad$

$$
\begin{aligned}
& 81=\left|225-b^{2}\right| \\
& b^{2}=225-81 \\
& b^{2}=144
\end{aligned}
$$

Substitute 0 for $h$ and $k$, the square root of 144 for $b$ and 15 for a into the standard form equation of an ellipse. The equation is $y$ squared over the square of 15 plus $x$ squared over the square of the square root of 144 equals tol.

$$
\begin{aligned}
& \frac{(y-h)^{2}}{a^{2}}+\frac{(x-k)^{2}}{b^{2}}=1 \\
& \frac{(y-0)^{2}}{(15)^{2}}+\frac{(x-0)^{2}}{(\sqrt{144})^{2}}=1 \\
& \frac{y^{2}}{(15)^{2}}+\frac{x^{2}}{(12)^{2}}=1
\end{aligned}
$$

## Explanation\#3

Step1) First we should see what we have find out.
Step2) $\frac{(x-h)^{2}}{\mathrm{a}^{2}}+\frac{(y-k)^{2}}{b^{2}}$
To write the equation in standard form, the center, $a^{2}$ and $b^{2}$ must be found. The $x$ coordinate of the vertices and foci given are 0 , so the center of the ellipse has to be ( 0,0 ).
|a| = horizontal distance
|b| = vertical distance
$(\mathrm{H}, \mathrm{k})=$ Center
$(0,0)=$ Center
Vertices: $(0,-10),(0,10)$
$\qquad$

Vertices are 20 units apart.
Foci: $(0,3),(0,-3)$
Foci are 6 units apart.
Since the foci are 10 units apart, indicating that $|c|$ is 3 and $c^{2}$, then is 9 .
$|c|=3 \quad c^{2}=3^{2}$
To find $b^{2}$, use the foci equation. $c$ is the distance from the center of the ellipse to the foci. Since the center is ( 0,0 ), and the vertices are ( $0,-10$ ) and $(0,10)$, a equals 10 .
$c^{2}=|a-b|$
$|C|=$ distance from center to foci
$(0,0)=$ Center
$(0,-10),(0,10)=$ vertices
$|a|=$ distance from center to vertex
$|a|=10$
Now that we have the values for a and c, we can substitute the values into the equation and simplify to find $b$.

$$
\begin{aligned}
& \mathrm{C}^{2}=\mid a^{2}-b^{2} \\
& (3)^{2}=\left|10^{2}-(b)^{2}\right| \\
& 9=\left|100-b^{2}\right| \\
& \mathrm{b}^{2}=100-9 \\
& \mathrm{~b}^{2}=91
\end{aligned}
$$

Substitute 0 for $h$ and $k$, the square root of 91 for $b$ and 10 for a into the standard form equation of an ellipse. The equation is $y$ squared over the square of 10 plus $x$ squared over the square of the square root of 91 equals tol.

$$
\begin{aligned}
& \frac{(y-h)^{2}}{a^{2}}+{\frac{(x-k)^{2}}{b^{2}}=1}_{{\frac{(y-0)^{2}}{(10)^{2}}}^{2}+\frac{(x-0)^{2}}{(\sqrt{91})}{ }^{2}=1}^{\frac{y^{2}}{(10)^{2}}+\frac{x^{2}}{(\sqrt{91})^{2}}=1}=1
\end{aligned}
$$

